

TAVUA COLLEGE
RESOURCE MATERIAL 2 2021
YEAR 13 MATHEMATICS

Instructions:

- Attempt the following questions daily on a separate sheet of paper.

Monday 21/06/21

1. Find x and y such that $x + 3i = 2 + yi$
2. Solve $5x^2 + 4x + 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. For the complex number, $z = -1 - \sqrt{2}i$,
 - (a) Write z in polar form.
 - (b) Find z^3 . Leave your answer in polar form.

Tuesday 22/06/21

1. Given $z = -1 - \sqrt{3}i$,
 - (a) Plot z and \bar{z} on an Argand diagram.
 - (b) Find $|z|$ and $\arg(z)$.
2. f and g are defined as $f: x \rightarrow x^2 + 1$ and $g: x \rightarrow \frac{1}{x-4}$

Find

- (a) $g^{-1}(x)$
 - (b) $g \circ f(x)$
3. Express $\frac{6+2i}{3-i}$ in the form $a+bi$
 4. Two complex numbers are given as $\alpha = 3 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$ and $\beta = 2 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$. Find $\alpha \beta$ in the form $a + bi$.

Wednesday 23/06/21

- Solve the equation $z^3 = -8i$ to obtain the three distinct roots. Leave your answers in rectangular form.
- The position vectors of points A and B are $\underline{\mathbf{a}} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ and $\underline{\mathbf{b}} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$
 - Determine scalar product of vectors a and b.
 - Find the angle between a and b.
- Find x and y such that $(2 - i)x + (1 + 3i)y = 7$.

Thursday 24/06/21

- What is the symmetric equation of the line through (2, -1, 4) in the direction of $\begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$.
- If A is the point (3, 4, -2) and B is the point (1, -2, -5), find the coordinate of point P on the line AB given that $\frac{AP}{PB} = \frac{3}{1}$.
- f and g are defined as $f: x \rightarrow x^2 + 1$ and $g: x \rightarrow \sqrt{x - 2}$
 - State the domain of g .
 - Find $f \circ g(x)$
 - State the domain of $f \circ g(x)$.
 - State the range of $f \circ g(x)$.
- Show that the two vectors $\mathbf{a} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ are orthogonal.

Friday 25/06/21

- Given $z_1 = 2 + i$ and $z_2 = 3 - 2i$
 - Find $|z_1|$ and $|z_2|$
 - Show that $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
- Find x and y such that $(2 + 3i)(x + yi) = 11 + 10i$
- Given $Z = -64i$, find:
 - $Re(Z)$
 - $Arg(Z)$
- Simplify and express $\frac{4-3i}{3+2i} + 3$ in the form of $a + bi$.

Monday 28/06/21

- For the complex number $z = -3 + \sqrt{3}i$,
 - Write Z in polar form.
 - Find z^4 using De Moivre's Theorem and express the answer in rectangular form.
- Given $W = 27cis \frac{-\pi}{2}$, represent W on an argand diagram.
- A is point $(-2, 5, 12)$ and B is the point $(13, -5, -18)$. If P is a point on the line AB such that $AP : PB = 1:4$, find the coordinates of P.
- A polynomial function is given as $y = f(x)$. Given that the function satisfies $f(0) = f(2) = f(5) = 0$ and $f(3) = 24$,
 - State the x and y intercepts of the function.
 - Find the equation of the function
 - Sketch the graph of the function.

Tuesday 29/06/21

- For the complex number $z = \sqrt{3} - i$, find
 - $\text{Re}(Z)$
 - $\text{Im}(Z)$
 - $|Z|$
 - \bar{Z}
 - $\arg(Z)$
- A polynomial function is given by $f(x) = (x + 2)(x - 1)^3(x - 3)^2$.
 - Find the x and y intercepts of the graph of $f(x)$.
 - Sketch the graph of $f(x)$ clearly showing the behaviour of the graph around the x intercepts.
- Solve $4x^2 + 9 = 0, x \in Z$.
- Express $\frac{1}{\sqrt{3}-i}$ in the $a + bi$.

Wednesday 30/06/21

- The polar form of a complex number Z is given as $8 \left(\cos \frac{-\pi}{2} + i \sin \frac{-\pi}{2} \right)$. Find the cube roots of Z and display the roots on an Argand diagram (give your answer in rectangular form).
- For the line $\frac{x+2}{3} = \frac{4-y}{2} = 2z + 4$,
 - State the coordinates of a point that lies on this line.
 - State a direction vector.
 - Write the vector equation of the line in parametric form.

3. A and B are the points $(-3, 5, 10)$ and $(12, -5, -15)$ respectively. Find the coordinates of a point P on line AB such that $\frac{AP}{AB} = \frac{2}{5}$. (Note the ratio carefully).
4. Solve the equation $4x^2 + 2x + 1 = 0$

Thursday 01/07/21

1. Functions f and g are defined as $f: x \rightarrow \sqrt{x}$ and $g: x \rightarrow x^2$. Find
- The domains of f and g .
 - $f \circ g(x)$ and its domain
 - $g \circ f(x)$ and its domain.
2. A complex number Z has $Re(Z) = -2$ and $Im(Z) = \sqrt{3}$. Find:
- \bar{Z} , the conjugate of Z , in rectangular form.
 - $Z \cdot \bar{Z}$
 - $\frac{1}{Z}$ in the form of $a + bi$
3. Plot the following on an argand diagram.
- $|z| < 2$
 - $-3 < Re(z) \leq 2$
 - $-1 < Im(z) \leq 2$
4. Solve for x and y in $(-2 + \sqrt{3}i)(x + yi) = 3 - 2i$

Friday 02/07/21

1. A polynomial function is given by $f(x) = -x^3 + 2x^2 + 5x - 6$.
- Given $f(1) = 0$, find the x and y intercepts of the graph of $f(x)$.
 - Sketch the graph of $f(x)$.
2. Given vector $p = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$ and vector $q = \begin{pmatrix} -5 \\ 2 \\ 4 \end{pmatrix}$, find the constant k such that $5p + kq = \begin{pmatrix} 20 \\ -14 \\ 7 \end{pmatrix}$
3. Find the unit vector in the direction of $p = 4i - 4j + 7k$.
4. Graph the complex number $z = 5 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$ on an Argand diagram.

THE END